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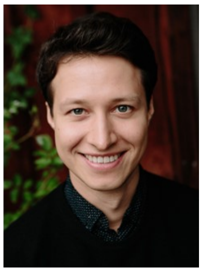
Conditional density forecasting: a tempered importance sampling approach*





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We introduce a new methodology to modify the forecasting densities of a BVAR and a DSGE model so that they can be adjusted to include financial markets information about future oil prices. Perceived future asymmetric upside risks to oil prices result in upside risks to headline and core inflation, while downside risks to GDP. A real-time forecasting exercise yields that introducing market-based information on the oil price improves inflation and GDP forecasts during crises times, such as the COVID pandemic.

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1. Introduction

In recent years policy makers realized the importance of understanding the risks surrounding their forecasts and how possible asymmetric risks arising in one part of the economy may propagate to the rest. An example could be the significant increase of financial markets' perceived risks to future energy prices stemming from the Russian invasion of Ukraine. Other example is related to disagreements by professional forecasters, households or firms about future developments and how those risks/disagreements may translate to inflation and economic activity. Incorporating this distributional information into the density forecasts produced by macroeconometric models is not trivial. While there exist several methods in the literature to condition model-based forecasting densities on external information about the future paths of some variables included in the model, less research considers conditioning those forecasting densities not only on the first but also on higher moments about their marginal distributions, such as the variance or skewness. Moreover, limiting the information set to an expected path of the model variables ignores the evolution of risks around these forecasts.

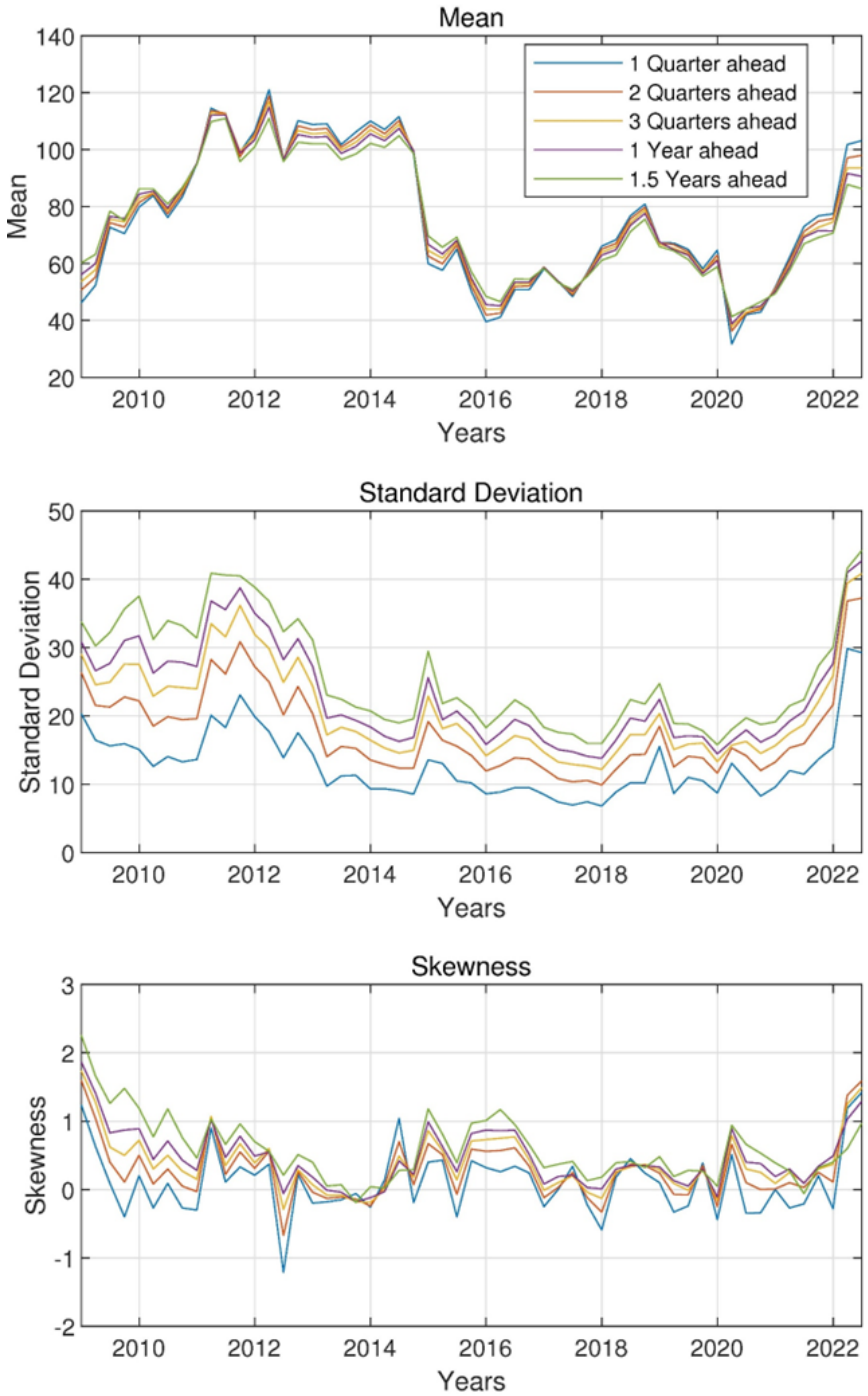
In a recent paper, we develop a new and robust methodology that allows to incorporate information available to policymakers or researchers to model-based density forecasts via entropic tilting. In a nutshell, our methodology is a modification of the tempered importance sampling approach developed by Neal (2001) and further refined by Herbst and Schorfheide (2014) and Herbst and Schorfheide (2019). Our methodology allows to slowly change the model-based density forecasts for one or more variables in a model to some other final target densities that incorporate external or conditioning information, while at the same time keeping track of how the distributions of other variables in the model change.

2. Application

To showcase how our algorithm works, we use our methodology to condition the density forecast of a small euro area BVAR model and a large DSGE model on option-implied densities of oil price futures. The idea is to introduce information on asymmetric oil price forecasting densities based on financial market data into the different models and to explore how the additional information tilts the forecasting distribution of other macroeconomic variables such as inflation and real GDP.

Figure 1 shows the first three moments of the option implied probability density functions of oil prices using quarterly data from 2008 to 2022 for different forecast horizons obtained from the ECB's Statistical Data Warehouse. Most notably, the probability density of the future oil price exhibits large fluctuations in the evolution of skewness for all horizons over the full sample, with increasing uncertainty for longer horizons. Remarkably, in 2022, with the beginning of the Russian invasion of Ukraine, market-based data shows that agents expected significant asymmetric upside risks to the price of oil. Our goal is to find how the incorporation of this external risk assessment translates to inflation and GDP in different models. Furthermore, we are interested if this information also improves the model-based forecasting accuracy.

Figure 1: Moments of the option-implied densities for future oil prices



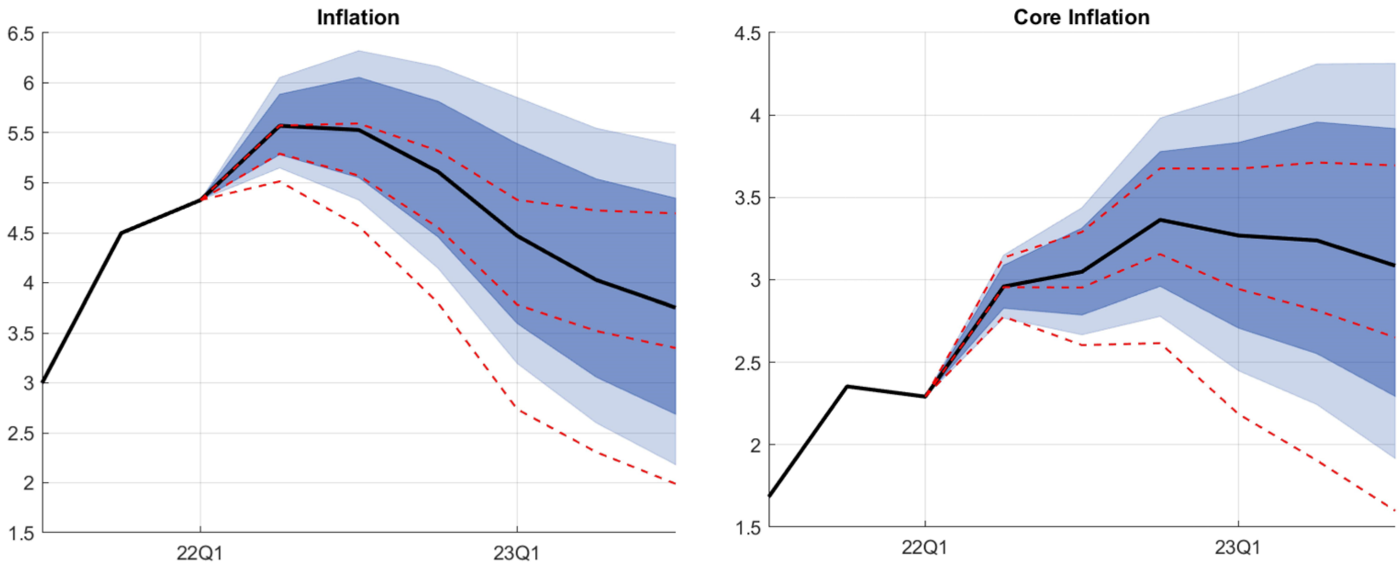
Notes: First 3 moments over time (2009 – 2022) implied from financial market options on oil prices futures.

As a first step, our methodology requires a parametric distribution to approximate the information that researchers would like to condition on, that can be feed into the tempered importance sampling algorithm. For the option-based densities of the future price of oil, we use a multivariate skew-t distribution. This is a very flexible distribution that allows us to capture the skewness in the distributions as well as possible outliers.

We then impose the skew-t distribution that we obtain in the first step on the forecasts of a BVAR model for the euro area. The model includes as observables the log of the price of oil, the log of real GDP, the log of prices including and excluding energy as well as the log of the US/Dollar exchange rate, log of employment and the long- and short-term interest rates. We first compute the model consistent density forecasts, including parameter uncertainty, and then tilt them such that the marginal forecasting distribution for oil prices matches the market-based ones.

As an example, Figure 2 shows the resulting densities for inflation and core inflation when we introduce the information of the option-implied densities on the 4th of March of 2022, once the Russian invasion of Ukraine had started. The shaded areas show the 16, 25, 75 and 84 percent quantiles of the resulting forecasting distribution of the year on year inflation rate together with the median given by the solid black line. Additionally, the dotted red lines show the 16 and 84 percent quantiles of the original model-based distributions. In both cases, introducing the information of the options results in an upward shift of the full distributions. In the case of inflation, the new median nearly coincides with the original 84 percent quantile of the original distribution in the first two periods. The forecasting density of core inflation is close to the original model in the first period but subsequently deviates from the original model with significantly higher values over the rest of the forecasting horizon. Additionally, the positive skewness in the distribution of the oil prices also results in upside risks to inflation.

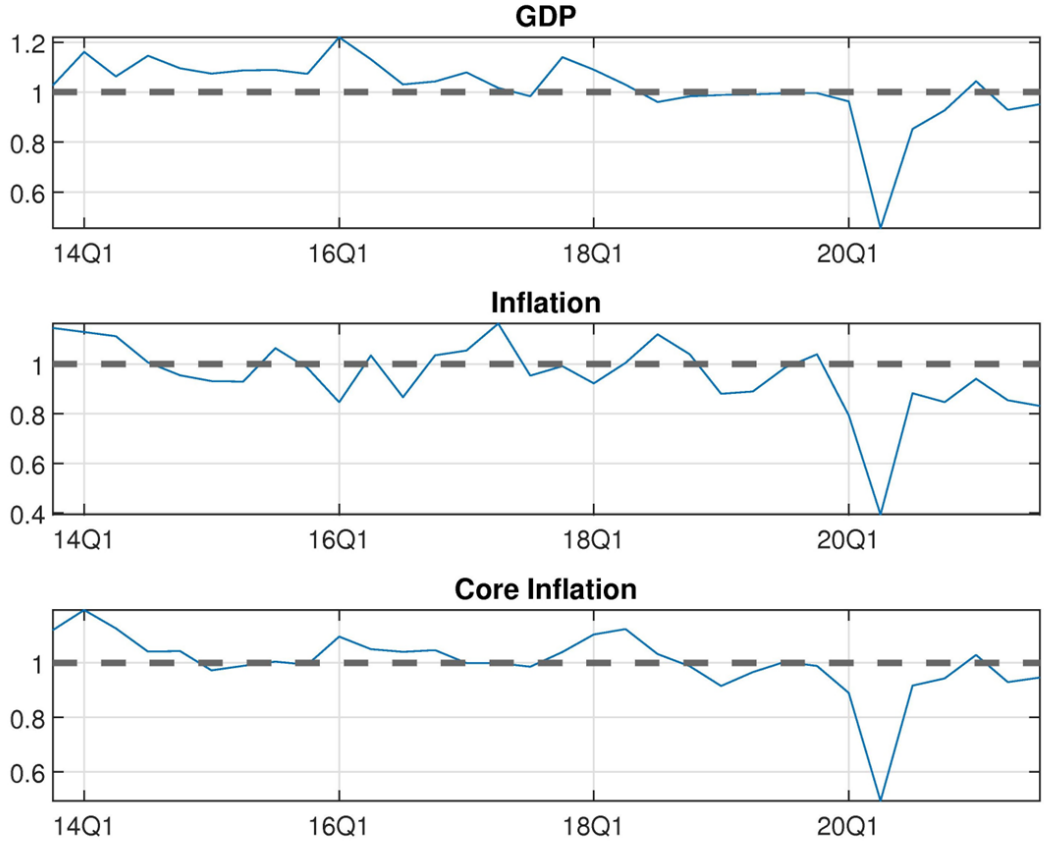
Figure 2: Impact of option-implied forecasting densities for oil price in a BVAR



Notes: BVAR forecasting densities for the year-on-year inflation rate (headline an core) before (16-84 percent quantiles in dotted red lines) and after (16-25-75-84 percent quantiles in blue) introducing option-implied densities of the 4th March 2022.

To further evaluate the effect of conditioning on information about the market-based forecasting distribution of our BVAR model, we also look at the probabilistic forecasting performance in a real time forecasting exercise to forecast GDP, inflation and core inflation. We estimate the same BVAR as before using data vintages starting in the last quarter of 2013 up until the third quarter of 2021. Since our methodology seeks to incorporate information about the full distribution, we use the continuous ranked probability score (CRPS) as the metric to evaluate the density forecasts. Figure 3 shows the ratio of the CRPS of the tilted densities compared to the original model-based densities. The results for inflation indicate that while including additional information on the distribution from options on oil prices does not increase predictive accuracy in moderate periods with stable economic conditions, it strongly increases the probabilistic forecasts accuracy in times of economic turmoil during the onset of the Covid pandemic in the first and second quarter of 2020.

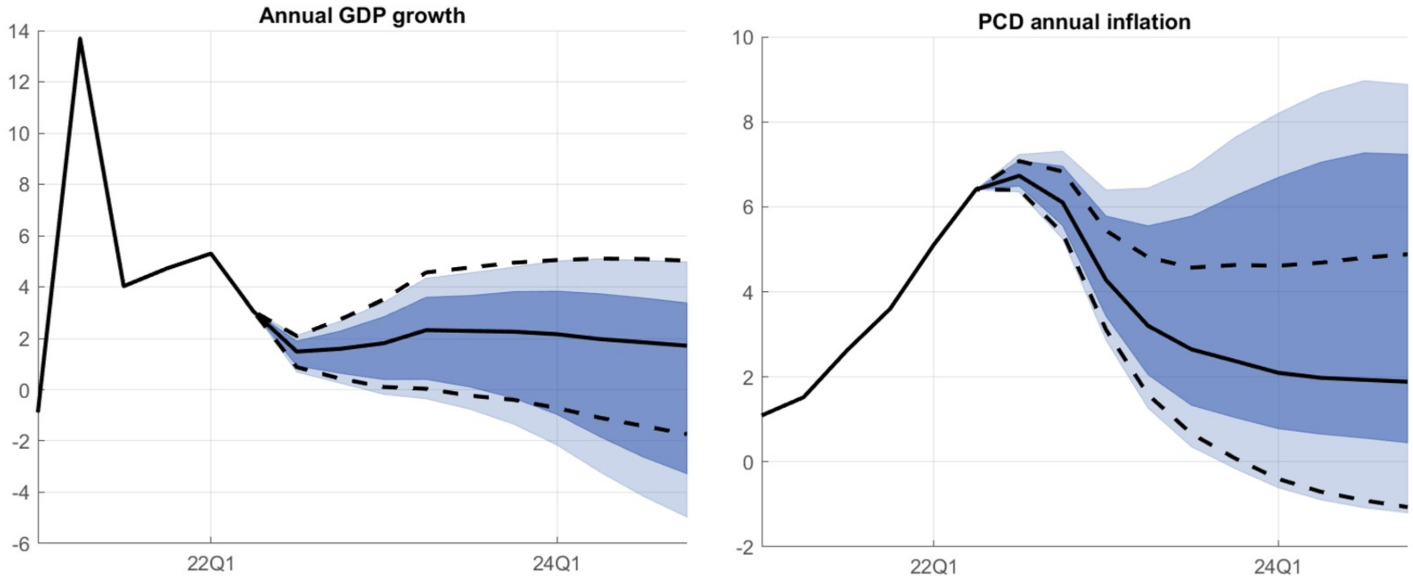
Figure 3: Continuous ranked probability score (CRPS) ratios



Notes: Ratio of the CRPS of the tilted densities compared to the original model-based densities.

Finally, our methodology can also be used in other types of models, such as DSGE models. We also impose the option-based densities in the NAWM model of the euro area (Coenen et al. 2018). Figure 4 shows the results of the exercise for GDP growth and inflation. Black lines show the model-based forecast, while the blue shaded areas represent the tilted distributions that match the distribution of the price of oil with the market-based measures. In the model, the price of oil behaves as a supply side shock. Thus, once we incorporate option implied information that assumes that the distribution of the price of oil is skewed to the upside, the transmission channel in the model indicates significant downside risks to the real economy captured by annual GDP growth, and upside risks to inflation, represented by inflation in the private consumption deflator. Figure 4 also shows that the final distributions are skewed, and the asymmetries are inherited from the skewness in the market-based options.

Figure 4: Impact of option-implied forecasting densities for oil prices in the NAWM II



Notes: NAWM II forecasting densities for GDP and inflation (private consumption deflator) before (16-84 percent quantiles in dotted black lines) and after (16-25-75-84 percent quantiles in blue) introducing option-implied densities of the 4th March 2022.

3. Conclusion

We develop a methodology that can be used to condition probabilistic forecasts of a model on off-model information about the marginal distributions of some of the model variables. The algorithm is based on a tempered importance sampling method. We show that the algorithm is flexible enough to be used for different models. Our methodology as well as our application is widely applicable and provides several extensions for further research such as introducing information from traded derivatives with other underlyings such as interest rates or exchange rates, or to introduce information about judgemental forecasts or disagreement about different forecasters.

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