Strike while the Iron is Hot:

Optimal Monetary Policy with a Nonlinear Phillips Curve

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Motivation

- The recent inflation surge featured
  - Increase in the frequency of price changes (Montag and Villar, 2023) US
  - Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US

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- Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)
- What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

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- Positive analysis under a Taylor rule
- Normative analysis: Ramsey optimal policy
  - Optimal long-run inflation
  - Characterize optimal responses to shocks

What we find

- Positive analysis:
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- Normative analysis:
  - When cost-push shocks are small, business as usual.
  - When cost-push shocks are large, more *hawkish* policy: "strike while the iron is hot."
  - Divine coincidence holds for efficient shocks, either small or large.
  - Optimal long-run inflation is slightly positive.
  - The time-inconsistency problem is there, but weakened relative to standard framework.

Literature

- Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)
  - Microfounded by state-dependent price setting

    (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)
  - In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)

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- Optimal policy in a menu cost economy
  - Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)
  - Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)
  - Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of (our version of) the Golosov-Lucas model

$=$ Textbook, Discrete-time New-Keynesian model with Calvo pricing (e.g. Galí, 2008)

– Calvo fairy [Calvoplus also includes this component]

$+$ fixed costs of price adjustments η

$+$ stochastic, idiosyncratic product quality $A_t(i)$

$=$ Heterogeneous-firm NK DSGE model.

Sketch of the model

- Households consume a Dixit and Stiglitz (1977) basket of goods, work and save.
- Per-period utility of consumption is log and disutility of labor is linear.
- Idiosyncratic quality $A_t(i)$ implies that

$$C_t = \left\{ \int \left[A_t(i) C_t(i) \right]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}.$$

- Monopolistic producers with $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, A_t is aggregate productivity.
- Firms face a fixed cost in labor units η to update prices and an employment subsidy τ_t.

Pricing decision

- Define $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log real price.
- Define $\lambda_t(p)$ be the price-adjustment probability. Value function is

$$
\begin{aligned}
V_t(p) &= \Pi(p, w_t, A_t, A_t(i), \tau_t) \\
&+ \mathbb{E}_t\left[(1-\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}))\,\Lambda_{t,t+1} V_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\right] \\
&+ \mathbb{E}_t\left[\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\,\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}(p') - \eta w_{t+1}\right)\right].
\end{aligned}
$$

- The price adjustment probability is characterized by a (s,S) rule:

$$\lambda_t(p) = I[\max_{p'} V_t(p') - \eta w_t > V_t(p)].$$

Monetary Policy and shocks processes

- For positive analysis only, monetary policy follows a Taylor rule:

$$\log\left(R_t\right) = \rho_r \log\left(R_{t-1}\right) + \left(1-\rho_r\right)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p) dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p) dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

where $g_t(p)$ is endogenous object.

The model in one slide

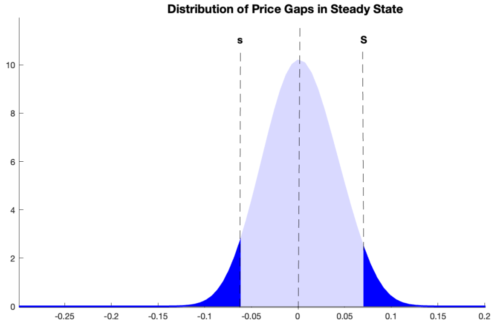
$$\max_{\{g_t^c(\cdot),g_t^0,V_t(\cdot),C_t,w_t,p_t^*,s_t,S_t,\pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon \frac{C_t}{A_t} \left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\, dx + g_t^0 e^{(p_t^*)(-\epsilon)} \right) - \upsilon \eta g_t^0 \right)$$

subject to

$$\begin{aligned}
1 &= \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\, dx + g_t^0 e^{(p_t^*)(1-\epsilon)}, \\
0 &= \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t,t+1} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial \phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x} dx' \\
&+ \Lambda_{t,t+1} \left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right)\right) (V_{t+1}(0) - \eta w_{t+1}), \\
V_t(s_t) &= V_t(0) - \eta w_t, \\
V_t(S_t) &= V_t(0) - \eta w_t, \\
w_t &= \upsilon C_t^{\gamma}, \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1} \frac{1}{\sigma} \int_{s_t}^{S_t} \left[V_{t+1}(x') \phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right] dx' \\
&+ \Lambda_{t,t+1} \left(1 - \frac{1}{\sigma}\int_{s_t}^{S_t} \left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right] dx' \right) [(V_{t+1}(0) - \eta w_{t+1})], \\
g_t^c(x) &= \frac{1}{\sigma} \int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1}) \phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma}\right) dx_{-1} + g_{t-1}^0 \phi\left(\frac{-x-\pi_t^*}{\sigma}\right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x) dx.
\end{aligned}$$

Model: Intuitive summary

- Each period, firm *i* chooses whether to reset its price and, if so, what new price to set
- The firm's optimality conditions define the reset price and the inaction region (S,s)



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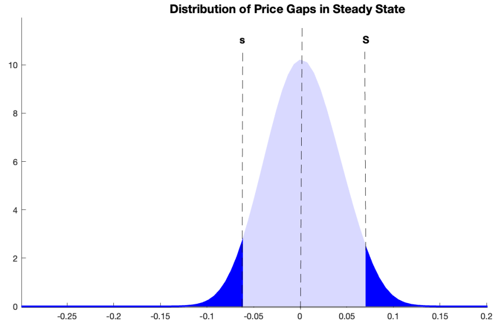

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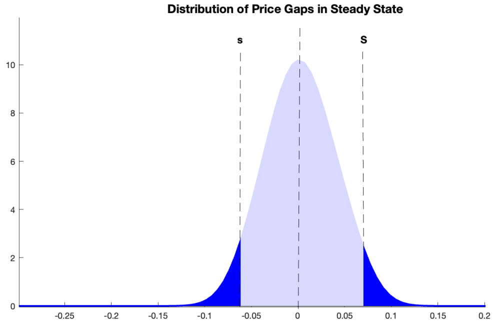
- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price



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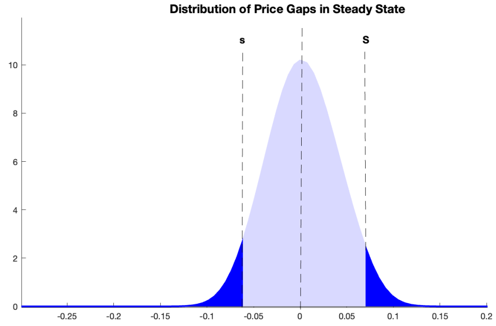
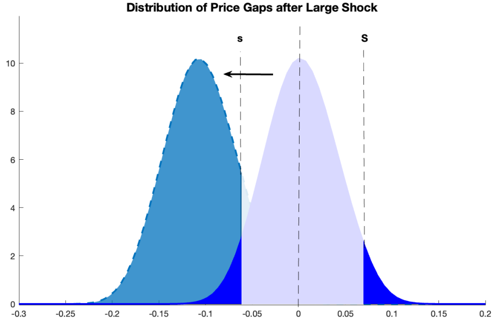
- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price
- Let $x_t(i) \equiv p_t(i) - p_t^*(i)$ be the difference of that price from the optimal price



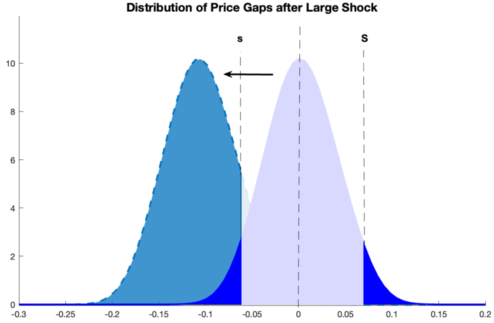
Model under large shock

- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands



Model under large shock

- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands
- Pushes a large fraction of firms outside of the inaction region
- Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of "selection")

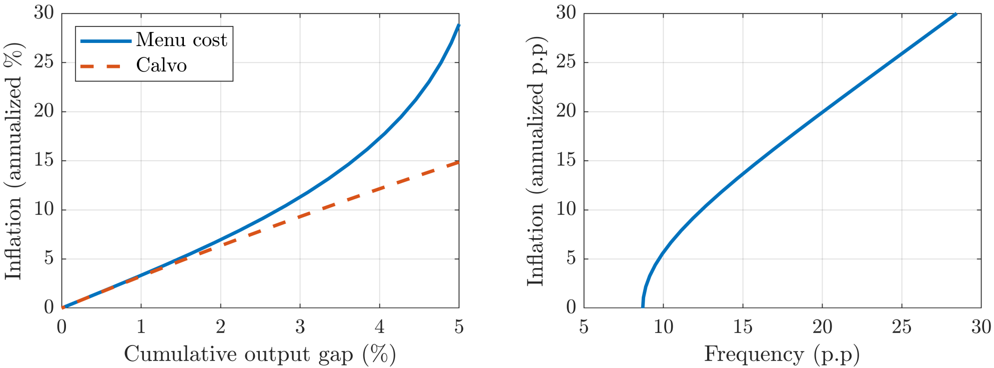


Calibration

Households			
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set so that $w = C$
Price setting targets			
Frequency	8.7%	Frequency of price changes	Nakamura and Steinsson (2008)
Size	8.5%	Absolute size of price changes	Nakamura and Steinsson (2008)
Monetary policy			
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.125	Output coefficient in Taylor rule	Taylor (1993)
Shocks			
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.25^{1/3}$	Persistence of the cost-push shock	

Main positive result: Non-linear Phillips curve

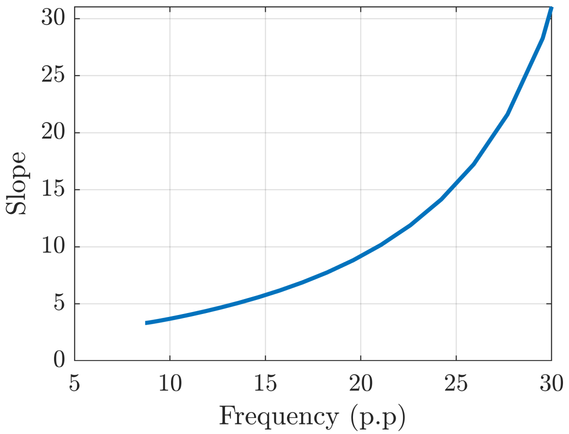
Small shocks: like *adjusted* Calvo; large shocks: non-linear. more



Corollary: State-dependent monetary policy

- P.C. slope determines the sacrifice ratio: the relative impact on inflation versus output gap of a marginal monetary policy tightening.

- Key: state-dependent monetary policy effects.



Normative analysis: Computation

- Challenges
  - Price distribution $g_t(p_t)$ and value function $V_t(p_t)$ are infinite-dimensional objects
  - We need sufficient accuracy for optimal policy assessment
- New algorithm, in discrete time
  - Approximate distribution and value functions by piece-wise linear functions on grid.
  - Endogenous grid points: (S,s) bands and the optimal reset price.
  - Evaluate integrals analytically.
  - Solve non-linearly in the sequence space using Dynare's perfect foresight Ramsey solver.

Normative result 1: Optimal response to cost-push shocks is non-linear

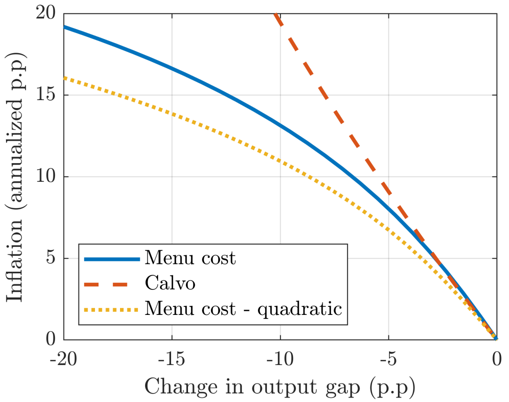
- In the textbook, LQ framework, optimal policy is a price-level targeting rule

$$\hat{p}_t = -\frac{1}{\epsilon}\tilde{y}_t^e$$

- For small cost-push shocks, optimal policy in the menu cost model is about the same.
- For large cost-push shock, strike while the iron is hot!

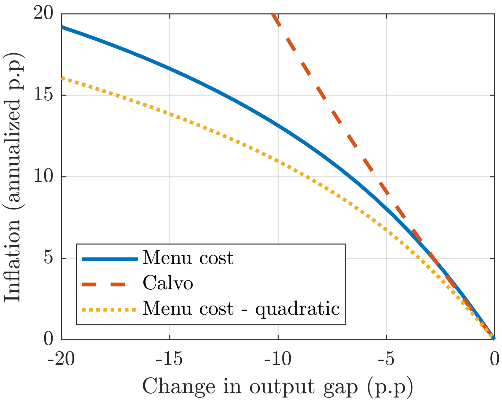
Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust



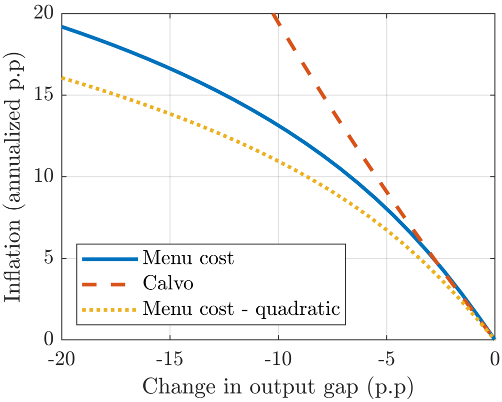
Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust
- After large shocks, the planner stabilizes inflation more relative to the output gap

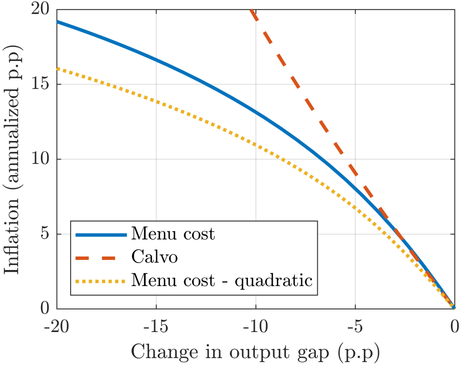


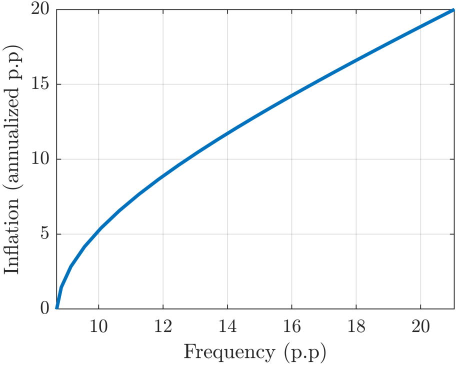
Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust
- After large shocks, the planner stabilizes inflation more relative to the output gap
- Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)
  - Similar results with quadratic objective
  - The nonlinearity of the targeting rule is due to the nonlinear Phillips curve

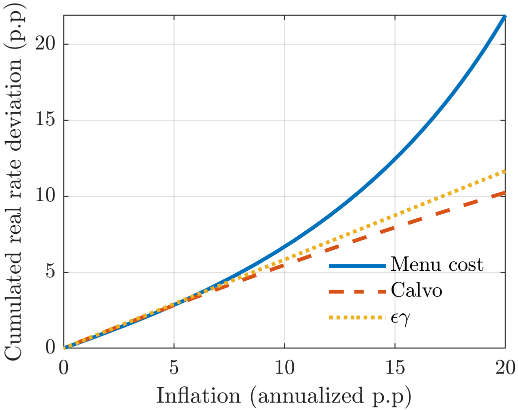


Nonlinear targeting rule



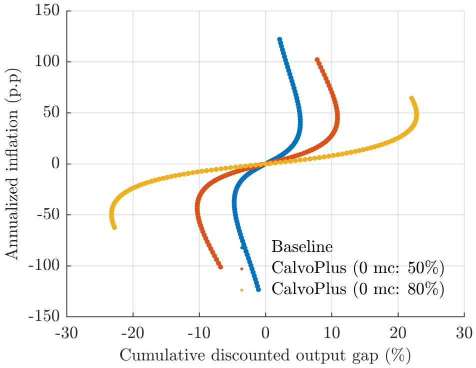


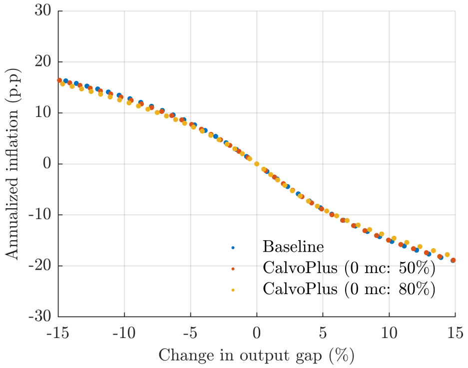
Nonlinear targeting rule for the real interest rate



Normative result 1.1: Calvoplus

Calvo plus: very different Phillips curve slope, almost the same optimal monetary policy.





Normative result 2: "Divine coincidence" holds

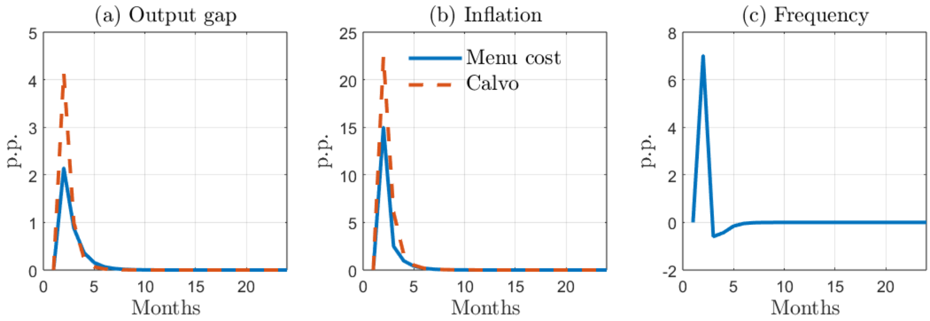
- In the standard NK model with Calvo pricing: divine coincidence holds after shocks affecting the efficient allocation: TFP (A_t) [also true for a discount rate shock].
- Optimal policy stabilizes inflation and closes the output gap.
- Same result holds in menu-cost models, regardless shocks are small or large.

Normative result 3: Optimal long-run inflation rate

- The steady-state Ramsey inflation rate is slightly above zero: $\pi^* = 0.3\%$
- Why not zero?
  - Asymmetric profit function: negative price gaps more harmful $=>$ Asymmetric (S,s) bands.
  - At zero inflation, more mass around the lower than higher threshold.
  - Slightly positive inflation raises p^* and pushes the mass of firms upwards.
  - $=>$ Lower frequency $=>$ less waste of resources paying for the menu cost.

Normative result 4: Time inconsistency is weakened by endogenous frequency

- Optimal policy without precommitment (time-0)
- Inefficient steady state
- Weaker time inconsistency in GL than in Calvo: costlier to increase output gap

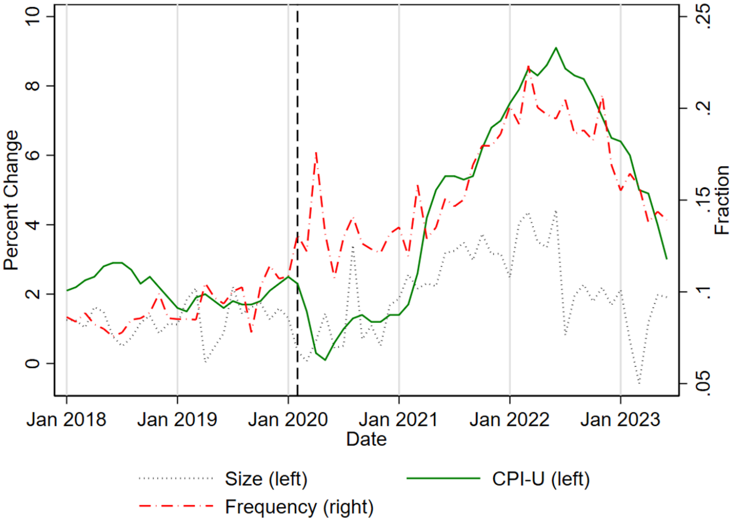


Conclusion

We study optimal policy in a menu cost model delivering a non-linear Phillips curve.

- Optimal response to small cost shocks similar to Calvo (1983).
- Lean against frequency for large cost-push shocks: strike while the iron is hot!
- Divine coincidence holds for efficient shocks, either small or large.
- Optimal long-run inflation is near zero.
- Time-inconsistency is there although weakened.

CPI and frequency of price changes in the US, Montag and Villar (2023)



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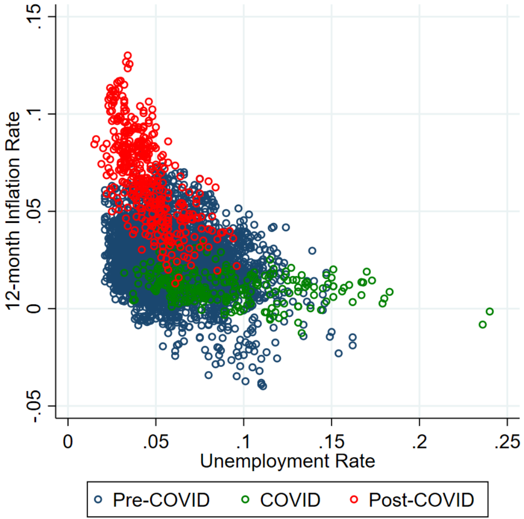
Phillips correlation across US cities, Cerrato and Gitti (2023)



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Modified Phillips correlation time, Benigno and Eggertsson (2023)

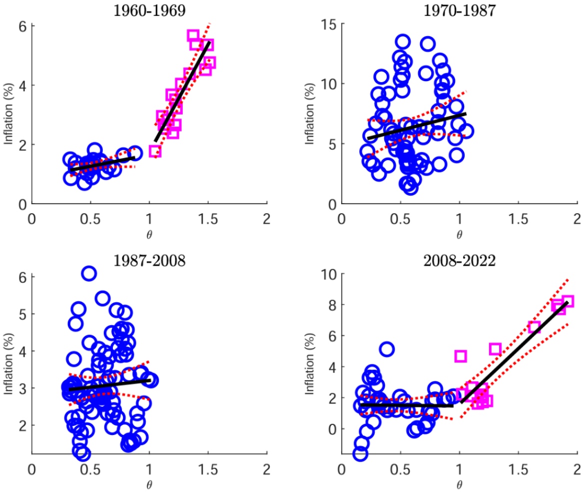

Figure 4: Inflation: CPI inflation rate at annual rates. θ: vacancy-to-unemployed ratio.

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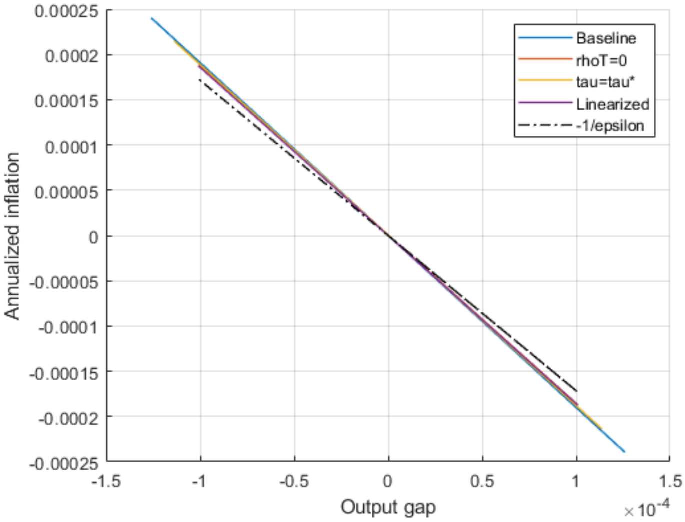
Slope of the target rule for small shocks



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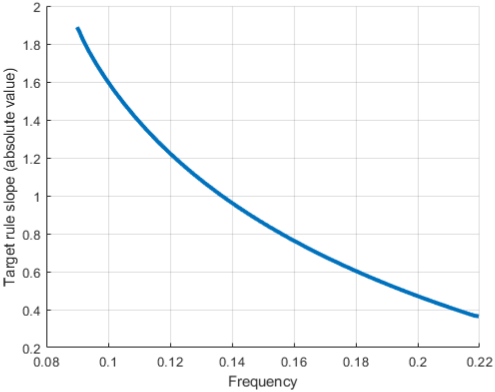
State-dependent inflation-output tradeoff

- Inflation-output tradeoff varies with frequency



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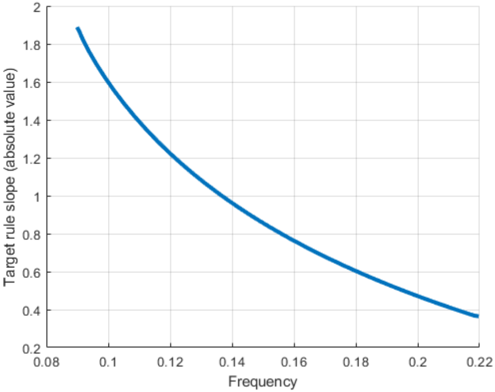
State-dependent inflation-output tradeoff

- Inflation-output tradeoff varies with frequency
- After large shocks, the planner stabilizes inflation relative to the output gap on the margin more Analogy with Calvo, 1983



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State-dependent inflation-output tradeoff

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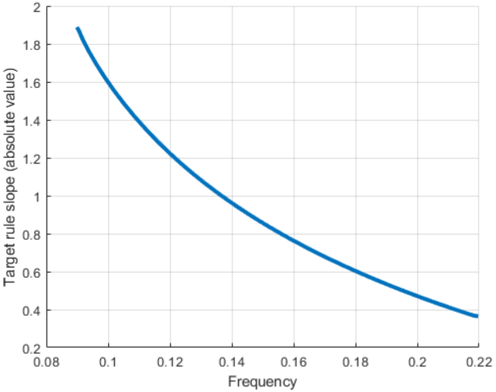
- Reduction in sacrifice ratio dominates decline in relative welfare weight of inflation



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Frequency and optimal policy in Calvo (1983)

- Optimal response to an iid cost-push shock (u_t)

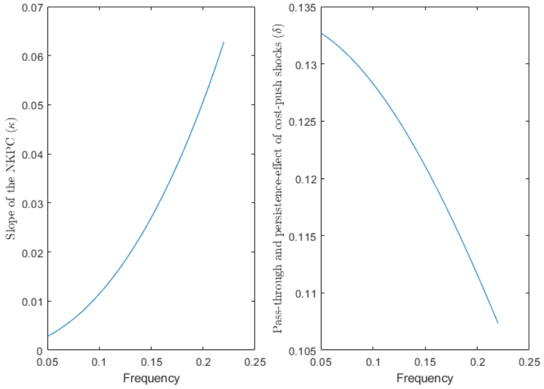
$$\hat{p}_t = \delta \hat{p}_{t-1} + \delta u_t$$

$$x_t = \delta x_{t-1} + \delta \epsilon u_t,$$

where $\hat{p}_t \equiv p_t - p_{-1}$ is the change in the price level and x_t is the output gap

- Parameter δ decreasing in frequency

- Reduction in sacrifice ratio dominates



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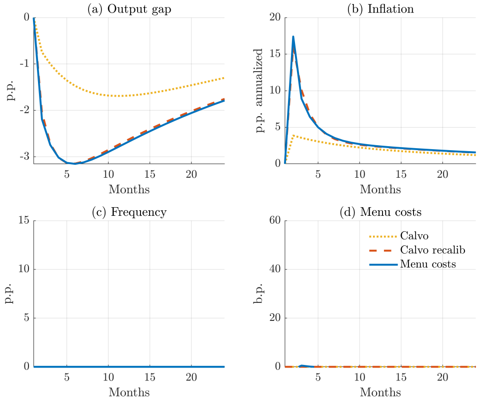
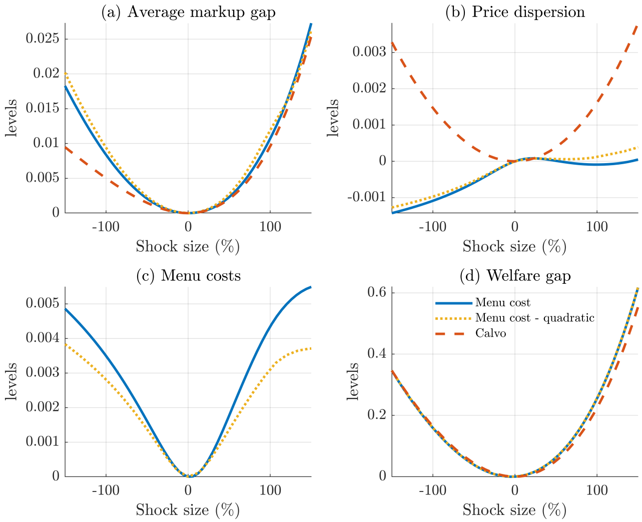
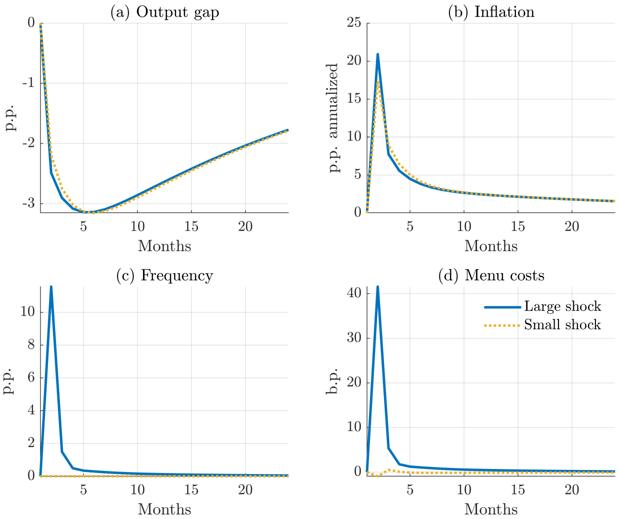
Response to a cost-push shock under a TR (Calvo vs. Golosov-Lucas)



Welfare decomposition

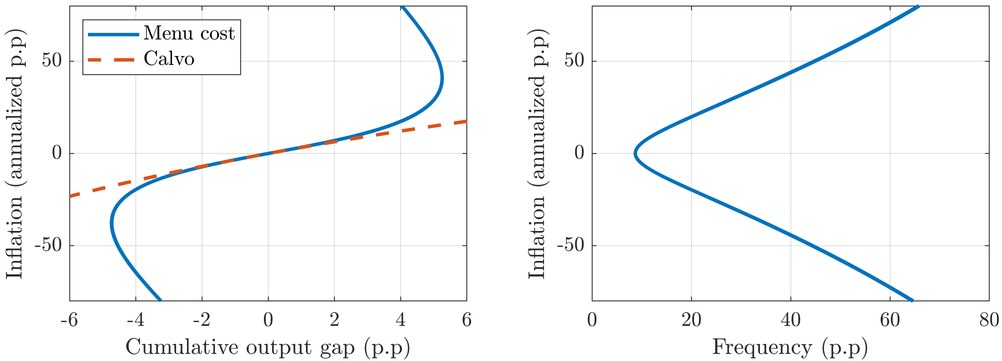


Response to a cost-push shock (large vs. small shock in Golosov-Lucas)



Main positive result: Non-linear Phillips curve

Small shocks: like *adjusted* Calvo; large shocks: non-linear, even bending backwards.



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